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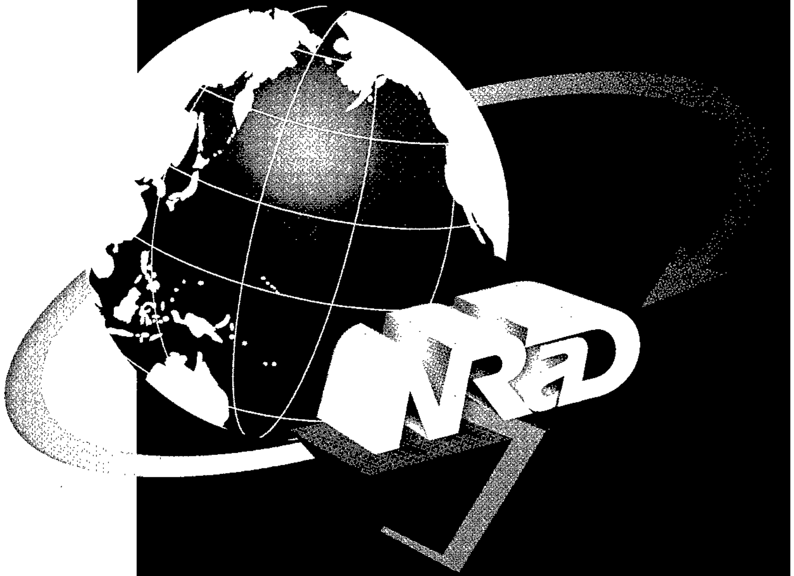

Dipole and Monopole Antenna Gain and Effective Area for Communication Formulas

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CONTENTS

Tables

INTRODUCTION

A comparison involving antenna measurements and performance predictions has sometimes revealed a 6-dB discrepancy between the ground-wave transmission measurements and the corresponding calculations. The source of this 6-dB discrepancy is attributable to an incorrect definition of the effective area of the receiving vertical monopole antenna. This erroneous definition is that the effective area of the vertical receiving monopole is twice as large as that of a corresponding receiving dipole in free space (Wolf, 1966). This report shows that the effective area of a monopole is one-half that of the corresponding dipole. It is this value of effective area that resolves the 6-dB discrepancy between measurements and analysis. This definition has meaning in the general forms of the transmission formula for communication and radar systems.

ANTENNA PARAMETERS

ANTENNA GAIN

The directivity and gain of an antenna are important measures of its performance as a transmitting system. If a transmitting antenna radiates isotropically, the power density or magnitude of the Poynting vector, P_{iso}, at a distance, r, from the transmitter is

$$P_{iso} = W_T / 4\pi r^2, \tag{1}$$

where W_T is the radiated power.

Antennas generally do not radiate power equally in all directions. The magnitude of the Poynting vector is not a constant, but is usually some function of the angles θ and ϕ or $P(\theta, \phi)$. The ratio of the power density radiated a fixed distance in a given direction to P_{iso} is the gain of the antenna. Thus, the transmitting gain describes the variation in power density with angel and is

$$G_T(\theta, \phi) = P(\theta, \phi) / P_{iso} . \tag{2}$$

Substituting equation (1) into equation (2),

$$G_T(\theta, \phi) = 4\pi r^2 P(\theta, \phi) / W_T . \tag{3}$$

If there are no losses in the antenna, the transmitter antenna gain is equal to the directivity of the antenna. If there are losses in the antenna, the gain is reduced by the radiation efficiency factor, k, from the directivity of the antenna, D_T, or

$$G_T(\theta, \phi) = k\, D_T(\theta, \phi). \tag{4}$$

ANTENNA EFFECTIVE AREA

The effective area measures the ability of a receive antenna to collect energy from an incoming wave. In general, the antenna effective area or effective aperture includes the effect of polarization and impedance mismatch. The antenna impedance is a measure of the ability of the antenna to act as a transducer between the propagation medium and the radio frequency (RF) system. The Thevinin-equivalent, two-terminal circuit for a receiving antenna system consists of a voltage generator and two impedances. The RF system loading the antenna is represented by the load impedance, $Z_L = R_L + jX_L$. The antenna is represented by a open-circuit voltage generator, V_A, induced in the receiving antenna by the impinging electromagnetic wave. The equivalent antenna impedance is $Z_A = R_A + jX_A$. When the loss resistance in the antenna is negligible, R_A is the radiation resistance of the receive antenna.

The current through the load impedance is given by

$$I = V_A / [(R_A + R_L)^2 + (X_A + X_L)^2]^{1/2}. \tag{5}$$

The available power, W_R, at the receiver load is

$$\begin{aligned} W_R &= I^2 R_L \\ &= V_A^2 R_L / [(R_A + R_L)^2 + (X_A + X_L)^2]^{1/2}. \end{aligned} \tag{6}$$

To obtain the maximum power available to the receive system load, the impedance of the system load must be equal to the complex conjugate of the impedance of the receive antenna. Thus, for this special case of maximum power transfer, the maximum available power at the receive system load is

$$W_{Rm} = V_A^2 / [4R_L]. \tag{7}$$

For any other matched condition, the transferred power will be smaller by the load mismatch loss factor, L_M:

$$L_M = R_L^2 / [(R_A + R_L)^2 + (X_A + X_L)^2]. \tag{8}$$

The received power, W_R, is also equal to the scalar product of the power density of the incident wave, $P(\theta, \phi)$, and the effective area, $A_e(\theta, \phi)$, of the receive antenna. The polarization mismatch is taken into account with the factor L_{POL}:

$$W_R = L_{POL}\, P(\theta, \phi)\, A_e(\theta, \phi). \tag{9}$$

The polarization mismatch factor is equal to 1 when the polarization of the incoming wave coincides with the antenna polarization. This is the condition for maximum receive power:

$$W_{Rm} = P(\theta, \phi)\, A_e(\theta, \phi). \tag{10}$$

Using equations (7) and (10), the maximum effective area is

$$A_{em}(\theta, \phi) = V_A^2 / [4R_L\, P(\theta, \phi)]. \tag{11}$$

ANTENNA EFFECTIVE HEIGHT

For linear or wire antennas, an antenna parameter similar to effective area is the concept of effective height. For the effective height of a receive antenna, it is sometimes convenient to express the induced open-circuit voltage, V_A, in terms of the incident peak field intensity, $E(\theta, \phi)$, and effective height, $h_e(\theta, \phi)$, of the antenna, or

$$V_A = E(q, f)\, h_e(q, f). \tag{12}$$

Again, the effective height is maximum when the polarization mismatch factor is 1. If the propagation medium has an intrinsic impedance, η,

$$P(\theta, \phi) = E(\theta, \phi)^2 / \eta. \tag{13}$$

Substituting equations (12) and (13) into equation (11), the maximum effective height is

$$h_{em}(\theta, \phi) = 2\,[R_L\, A_{em}(\theta, \phi) / \eta]^{1/2}. \qquad (14)$$

The concepts of effective area and effective height are intrinsically related.

TRANSMISSION EQUATION

Equation (3) can be rearranged to solve for the power density available at the receive antenna:

$$P(\theta, \phi) = G_T(\theta, \phi)\ \ W_T / 4\pi r^2. \qquad (15)$$

Considering equations (9) through (11) and equation (15), the transmission between the receive and transmit antenna can be derived by

$$W_R / W_T = G_T(\theta, \phi)\, A_{em}(\theta, \phi) / 4\pi r^2. \qquad (16)$$

The maximum effective area, $A_{em}(\theta, \phi)$, of a receive antenna can be related to a receive antenna gain by the following expression (Kraus, 1950):

$$G_R(\theta, \phi) = 4\pi\, A_{em}(\theta, \phi) / \lambda^2, \qquad (17)$$

where λ is the wavelength of the transmitted signal. In this formula it is assumed that radiation efficiency factor, k, of equation (4) is equal to 1. Substituting equation (17) into equation (16), the transmission formula for a communication circuit becomes

$$W_R / W_T = G_T(\theta, \phi)\, G_R(\theta, \phi)\, L_R, \qquad (18)$$

where the propagation loss is

$$L_R = \lambda^2 / (4\pi r)^2. \qquad (19)$$

TRANSMISSION FOR SEVERAL ANTENNA CONFIGURATIONS

The transmission equations are now applied to the following configurations of transmit and receive antennas.

DIPOLE TO DIPOLE IN FREE SPACE

Consider two dipoles in free space that are oriented parallel to each other and situated a distance of 100 wavelengths apart. At 100 wavelengths, the two antennas are in the far field of each other. Both dipoles are half-wave in length and have a radius of 0.0001 wavelengths. One dipole antenna is a center-fed transmitting dipole. The other antenna is a receiving dipole that is loaded in the center with a conjugate matched load.

The half-wave dipole gain is 2.16 dB (Kraus, 1950). Since the receive dipole has a conjugate load impedance, equation (18) can be used to calculate the transmission between the dipoles. These calculations can then be verified by numerical computation using the method of moments. For this analysis, *MININEC Professional* is used (Rockway et al., 1995).

MININEC Professional is a frequency-domain, method-of-moments computer code for the analysis of wire antennas. Using the method of moments, *MININEC Professional* solves an electric field formulation for the currents on electrically thin wires using a Galerkin procedure with triangular basis functions. Radiation patterns, near fields, charge distribution, impedance and other useful parameters are computed from the current solution. This formulation results in an unusually compact and efficient computer algorithm. In this newest *MININEC* effort, the formulation has been changed to use triangular basis functions, resulting in greater accuracy. The limitations of previous versions of *MININEC* have been eliminated. The computational engines are written in FORTRAN for greater speed and to make maximum use of available memory to set array sizes. The user interface runs under the *Microsoft Windows* environment. This represents a significant improvement in the MININEC antenna modeling capability. The user interface provides many convenient options for defining antenna geometry. Built-in graphics, online context-sensitive help, and problem diagnostics aid the user in problem definition. Output graphics and linkage to spreadsheets and word processors of other *Windows* applications greatly enhance the interpretation and analysis of *MININEC* computations.

The results of *MININEC Professional* and equation (18) for the free-space, half-wave dipole antenna problem are given in table 1. The *MININEC Professional* results verify the use of equation (18) and the half-wave dipole antenna gain of approximately 2.17 dB.

Table 1. Half-wave dipole antenna configuration (parallel dipoles in free space).

	Theory (dB)	*MININEC Professional* (dB)
Gain	2.16 (Kraus)	2.17
Transmission	-57.7 (Eq. 18)	-57.6

MONOPOLE TO MONOPOLE ON A CONDUCTING SURFACE

Consider two vertical monopoles on an infinite, perfectly conducting ground plane and situated a distance of 100 wavelengths apart. Both monopoles are quarter-wave in length and have a radius of 0.0001 wavelengths. Again, *MININEC Professional* can be applied. Again, the antennas are thin and are in the far fields of each other. One antenna is a bottom-fed transmitting monopole. The other antenna is a receiving monopole that is base-loaded with a conjugate matched load.

The gain of a hemispheric source is twice the gain of the corresponding isotropic source, a source radiating in all directions (Kraus, 1950). Thus, the gain, $G_{TM}(\theta, \phi)$, of a transmitting monopole has twice the gain, $G_{TD}(\theta, \phi)$, of the corresponding transmitting dipole or

$$G_{TM}(q, f) = 2\ G_{TD}(q, f). \qquad (20)$$

Thus, the gain of a quarter-wave monopole should have twice the gain of a corresponding half-wave dipole, or 5.16 dB.

MININEC Professional is again used to verify the assumed gain of the monopole. *MININEC Professional* is also used to verify the use of equation (18) in the calculation of transmission between the transmit and receive monopoles. These results are summarized in table 2.

Table 2. Quarter-wave monopole antenna configuration (parallel monopoles on a ground plane).

	Theory (dB)	***MININEC Professional* (dB)**
Gain	5.16 (Kraus)	5.17
Transmission	-51.7 (Eq. 18)	-57.6

MININEC Professional and Kraus (1950) agree on the gain of the transmitting antenna. As indicated in table 2, the transmission results of equation (18) and *MININEC Professional* do not agree. However, the transmission prediction of equation (18) is approximately 6 dB or four times greater than the calculation of *MININEC Professional.*

Comparing the *MININEC Professional* results of tables 1 and 2, the transmission for the two dipoles in free space is the same as the transmission for the two equivalent monopoles on a conducting surface. Again, the gain of a transmitting monopole is twice the gain of the equivalent transmitting dipole. Considering equation (18) and the equivalence of the dipole problem and monopole problem transmission, the relationship of the effective area of a monopole, $A_{emM}(\theta, \phi)$, to the effective area of an equivalent dipole, $A_{emD}(\theta, \phi)$, is

$$A_{emD}(\theta, \phi) = 4\ A_{emM}(\theta, \phi). \qquad (21)$$

The effective area of the receive monopole is one-fourth the effective area of the corresponding receive dipole. Thus, if a monopole and a dipole are immersed in identical fields, the dipole will deliver to a matched load four times the power as is available from a monopole.

For the dipole, equation (17) is rewritten as

$$G_{RD}(\theta, \phi) = 4\pi\, A_{emD}(\theta, \phi) / \lambda^2. \qquad (22)$$

Substituting equations (20) and (21) into equation (17) for a monopole,

$$G_{RM}(\theta, \phi) = 16\pi\, A_{emM}(\theta, \phi) / \lambda^2. \qquad (23)$$

Using equation (23) in equation (16), the transmission between monopole antennas becomes

$$W_{RM} / W_{TM} = G_{TM}(\theta, \phi)\, G_{RM}(\theta, \phi)\, L_R / 4. \qquad (24)$$

MININEC Professional is used to verify the use of equation (24) in the calculation of transmission between transmit and receive monopoles. These results are summarized in table 3. The *MININEC* result verifies the use of equation (24) for the transmission between transmitting and receiving monopoles.

Table 3. Quarter-wave monopole antenna configuration (parallel monopoles on a ground plane).

	Theory (dB)	***MININEC Professional* (dB)**
Gain	5.16 (Kraus)	5.17
Transmission	-57.7 (Eq. 24)	-57.6

MONOPOLE TO DIPOLE ABOVE A CONDUCTING SURFACE

Consider a quarter-wave monopole mounted vertically on an infinite, perfectly conducting ground plane. The monopole is a bottom-fed transmitting antenna. A half-wave, receiving dipole antenna is situated 100 wavelengths from the transmitting monopole and is center-loaded with a conjugate matched load. The receiving dipole antenna is oriented vertically to the ground pane and is situated at various elevations above the ground plane. These elevations are measured from the bottom of the dipole and include 0.5, 1, 3, 5, 7, and 9 wavelengths. Both antennas have a radius of 0.0001 wavelengths.

Extrapolating the results of the previous sections, equation (16) can be adapted to calculate the transmission. For this configuration, $G_T(\theta, \phi)$ becomes $G_{TM}(\theta, \phi)$ for the transmitting monopole. Equation (21) is used for the receiving dipole with a gain of $G_{TD}(\theta, \phi)$. For this configuration, the transmission equation becomes

$$W_{RD} / W_{TM} = G_{TM}(\theta, \phi)\, G_{RD}(\theta, \phi)\, L_R. \qquad (25)$$

Equation (25) gives a transmission value of -54.7 dB for this configuration. The *MININEC Professional* calculation is also -54.7 dB for all of the receive dipole elevations.

DIPOLE TO MONOPOLE ON A CONDUCTING SURFACE

Consider a half-wave, transmitting dipole situated vertically at various distances above an infinite, perfectly conducting ground plane. These elevations are measured from the bottom of the dipole and include 0.5, 1, 3, 5, 7, and 9 wavelengths. The half-wave dipole is a center-fed transmitting antenna. A quarter-wave, receiving monopole is situated 100 wavelengths from the transmitting dipole and is bottom-loaded with a conjugate load. Both antennas have a radius of 0.0001 wavelengths.

Equation (16) can again be used to calculate the transmission. For this configuration, the existence of the ground plane results in a double increase in the field at the monopole over a free-space field from the dipole. Equation (23) is used for the receiving monopole with a gain of $G_{TM}(\theta, \phi)$. For this configuration, the transmission equation becomes

$$W_{RM} / W_{TD} = 2\, G_{TD}(\theta, \phi)\, G_{RM}(\theta, \phi)\, L_R. \tag{26}$$

Equation (26) gives a transmission value of -54.7 dB for this configuration. The *MININEC Professional* calculation is also -54.7 dB.

DIPOLE TO DIPOLE ABOVE A CONDUCTING SURFACE

Consider a half-wave dipole situated vertically at various distances above an infinite, perfectly conducting ground plane. The half-wave dipole is a center-fed transmitting antenna. A half-wave receiving dipole is situated 100 wavelengths from the transmitting dipole and is center-loaded with a conjugate load. The receive dipole has its center at 5.25 wavelengths above the ground plane. Both antennas have a radius of 0.0001 wavelengths.

Again, because of the existence of the ground plane, the transmitting dipole has a gain at the receiving dipole that is dependent on the geometry of the transmitting and receiving dipoles and their elevations above the ground plane. Equation (16) can again be modified to calculate the transmission for this configuration. $F_T\, G_{TD}(\theta, \phi)$ is the gain of the transmitting dipole antenna modified by the ground plane. Equation (22) is used for the receiving dipole with a gain of $G_{RD}(\theta, \phi)$. For this configuration, the transmission equation then becomes

$$W_{RD} / W_{TD} = F_T\, G_{TD}(\theta, \phi)\, G_{RM}(\theta, \phi)\, L_R. \tag{27}$$

To test equation (27) the bottom of the transmitting dipole is situated at 0.5, 1, 3, 5, 7, and 9 wavelengths above the ground plane. The values for $F_T\, G_{TD}(\theta, \phi)$ in equation (27) are calculated with *MININEC Professional.* Table 4 presents the transmission calculations of equation (27) and *MININEC Professional. When the MININEC Professional* calculations for $F_T\, G_{TD}(\theta, \phi)$ are used in equation (27), *MININEC Professional* verifies the transmission calculation of equation (27).

Table 4. Half-wave dipole antenna configuration (parallel dipoles above a conducting surface).

Transmitter Height	$F_T\ G_{TD}(q, f)$ (dB)	Eq. (27) Transmission (dB)	*MININEC Professional* Transmission (dB)
.5	7.8	-52	-52
1	7.3	-52.5	-52.5
3	1.8	-58	-58
5	-8	-67.9	-67.9
7	5.3	-54.6	-54.6
9	8	-51.9	-51.9

SUMMARY

In the above discussion a number of different antenna configurations have been considered. From these studies a general form of the transmission formula for a communication circuit can be derived. This general form is

$$W_R / W_T = F_T \, G_T(\theta, \phi) \; F_R G_{RM}(\theta, \phi) \; L_R. \tag{28}$$

To use this equation, the gain of a monopole is twice the gain of the corresponding dipole (i.e., equation (19)). When the transmitting dipole is in free space or the transmitting antenna is a monopole, F_T is unity. When the transmitting antenna is a dipole antenna and is located above a ground plane, F_T may not be unity and will depend on the elevation of the transmitting antenna above the ground plane and the location at which the radiated field is determined (e.g., the location of the vertical receive antenna). If the receive antenna is a monopole, F_R will be 0.25. If the receive antenna is a dipole oriented above a ground plane, F_R will be unity.

The maximum effective area of a receive monopole is one-fourth the maximum effective area of the corresponding dipole. The relationship of gain to maximum effective area for a dipole is

$$G_{RD}(\theta, \phi) = 4\pi \, A_{emD}(\theta, \phi) / \lambda^2. \tag{29}$$

The relationship of gain and maximum effective area for a corresponding monopole is

$$G_{RM}(\theta, \phi) = 16\pi \, A_{emM}(\theta, \phi) / \lambda^2. \tag{30}$$

The concepts of effective area and effective height are intrinsically related by

$$h_{em}(\theta, \phi) = [2 \, R_L \, A_{em}(\theta, \phi) / \eta]^{1/2}. \tag{31}$$

The maximum effective height of a receive monopole is one-half the maximum effective height of the corresponding dipole.

REFERENCES

Kraus, J. D. 1950. *Antennas.* McGraw-Hill Book Co., New York, NY.

Rockway, J. W. and J. C. Logan. 1995. *MININEC Professional for Windows.* EM Scientific, Inc., Carson City, NV.

Wolff, E. A. 1966. *Antenna Analysis.* John Wiley and Sons., New York, NY.

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